Can a mimicking synthetic equity structure dominate the risk return profile of corporate bonds?

PRELIMINARY DRAFT
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This version: December 31st 2017 (Preliminary draft)

Abstract

This paper proposes an empirical study that compares the investment profile of corporate bonds and an equity derivative barrier structure that aims at replicating the behavior of a corporate bond portfolio. In this respect, we compare the yield of both corporate bond indices and a synthetic equity derivative structure that mimics a corporate bond. This paper analyses it for various ratings, on the bond side, and various barrier levels for the synthetic equity structure, on the other side. We also propose recovery rate variations for each barrier level. The yield offered by the structure results to be above that of its related corporate bond indices at a given rating level. This is challenged then by comparing the probability of default of both products. For some high rating classes and corresponding low probabilities of default, the derivative structure offers a better "spread/default" profile than that of corporate bonds.

KEYWORDS: equity derivatives, synthetisation, corporate bonds, rating, probability of default

JEL classification: G11, G12

1 Introduction

Most academic articles on derivatives in asset management mainly offer 2 types of researches. The first one is about the use of derivatives by mutual fund. Koski & Pontif (1995) analyze how mutual funds use derivatives and the impact of using derivatives on performance and return distributions of traditional portfolios with stock and bonds. In this field of research, the derivative is seen as a hedging instrument of equity risk.The second one is a large field about path return of hedge funds and the impact of derivatives use on the hedge fund returns. Some investigate the motivation of hedge funds to use derivatives (see Chen & Tech, 2008), some other large literature provides hedge funds return analysis to draw conclusions on how derivatives can impact the efficient frontier with classical mean-variance or mean-higher moments analysis. These papers often take derivatives strategy as a whole and without any cross asset framework.

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Merton (1974) initiated the modern analysis of the link between equity and bond via options. He pointed out that the bondholder of a corporate bond has the equivalent of a short put position on equity firm. Campbell and Taksler (2002) initiate a deep analysis on the link between equity volatility and credit spread for a sample of individual firms.

In this paper we compare corporate bond indexes with a synthetic equity derivative structure that aims to mimic a corporate bond. This could have several interests. Firstly in a portfolio management perspective: does a replication of a corporate bond portfolio with equity options improve the investment profile by increasing the yield or decreasing the risk? Secondly, it is a way to go deeper in the testing of Merton framework on the corporate bondholder risk return profile continuity. The corporate bondholder has a bond, when everything is going well, and is short of a in the money put options when the firm asset decreases and approaches the liabilities level. The aim of our paper is to mimic the implicit equity optionality of corporate bond with a synthetic equity derivatives structure.Finally, it is a way to test the existence of a market continuum. The hypothesis of the market continuity stipulates that every risk return profile can find an asset class corresponding to this profile. Market continuity can also have an impact on market stability issues (see Hass, F, 2003).

This paper proposes an empirical study that compares the profile of corporate bonds and an equity structure that aims to mimic or replicate corporate bonds portfolio. Our empirical study compares the yield of corporate bond index and the yield offered by an equity derivatives structure that replaces the credit spread by a short equity digital put premium. Contrary to a vanilla put option in which the payoff is the difference between strike price and spot price, the the digital put option payoff is a pre-fixed amount. This pre-fixed amount id paid if a pre-defined barrier level is observed. This barrier can be observed only at maturity (european barrier), continuously during the entire life of the option (american barrier) or at some pre-defined dates (bermudean barrier) Many authors point out that the volatility is not an appropriate measure with asymmetric assets like options. Our study takes as risk measurement the probability of default for corporate bonds and its equivalent for the equity derivative structure. The risk return analysis is provided with a buy and hold principle. We don't provide any mark to market analysis.

The next section of this paper describes the synthetic equity structure and the pricing model we use to price it. Then the third section provides a data overview as used. The last section presents results before the conclusion.

2 Synthetic equity structure and pricing model

The synthetic equity structure we use is very close to a corporate bond in terms of expected payoff. With a classical corporate bond, the investor pays 100% of the nominal value at the issue date and then receive a periodic coupon. The coupon is a fixed percentage of the nominal value. The coupon is equivalent to the yield when the price is 100%. The yield received by the investor remunerate the credit risk taken by the investor. The synthetic equity structure we choose shall mimic a corporate bond expected payoff by transforming the credit risk by an equity asymmetric risk.

The synthetic equity structure has a 5 years maturity and pays coupon on annual basis. We choose a 5 years maturity for two reasons. The first one is to be the closest on the average duration of corporate bonds in order to better compare the results. Secondly for available data reason: the implied volatility dataset goes until a 5 years maturity options.

Table 1
Average duration by rating
This table shows the average duration for the different rating on the observed period

AA	A	A	BBB	B
5,74	5,10	5,28	5,57	5,74

As with a classical bond, the coupon is a percentage of the nominal value. In our synthetic equity structure the coupon is subject to a condition on an equity index evolution. A barrier index level is fixed at the issue date. The coupon will be paid in the following years until the maturity if the index level is above or equal to the barrier level.

The following parameters of the structure are fixed at the issue date:

- The annual coupon that will be paid to the investor if the equity index observed level is above the barrier level.
- The barrier level as a percentage of the closing price of the index at the issue date.
- The recovery rate: the percentage of the nominal value that is paid to the investor in case of index level is below the barrier level at coupon payment date. In case of the index being below the barrier level at coupon payment date, the synthetic equity structure stops and the investor receive an amount equivalent to the recovery rate multiplied by the nominal amount.

Each year at the annual coupon payment date, the underlying level is observed and two events can occur:

- 1st alternative: the underlying level is above the barrier level, the coupon is normally paid and the structure continues.
- 2nd alternative: the underlying level is below the barrier level and the structure stops. Instead of the coupon, the investor gets back the fixed recovery amount (nominal value multiplied by the recovery rate).

The same 2 alternatives occur at maturity date. If the underlying level is above the barrier level, the investors get back 100%. If the underlying level is below the barrier level, the investor receives the recovery amount.

Let's take the example of a 50% barrier with an underlying index at 3000 points at the issue date. The barrier index level is fixed at 1500 points (3000 x 50%). The coupon is fixed to 5% with a recovery rate of 40%. One year after the issue date, the index level is observed. If the index level is above 1500 points, the investor received 5% coupon and the structure keeps going. If the index level is below 1500 points, the structure stops and the investor received 40% of the nominal value. If the observed level at each observation date is above the barrier level, the investor receives 5 times the 5% annual coupon and the nominal amount at the end of 5th year. If at one of the 5 observation date, the index level is below the barrier level, the structure stops and the investor only receive 40% of the nominal value and keep the coupon received preceding years.

The synthetic equity structure we use is very similar to a classical corporate bond except 2 main differences. Firstly the synthetic structure barrier is not continuous as the index level is observed every year (Bermudean barrier). It means that the index level could be below the barrier level all the year before the observation date and go above only at the observation date. This is a difference for classical bond which can be in default during all the year continuously (as American barrier). Second difference is the recovery amount payment date. For a classical bond, the recovery amount is often paid several years after the event date of default. The synthetic equity structure pays the recovery amount at the event date of default. These two differences have an impact on the risk profile comparison between both financial products. The non-continuous barrier of the structure make it less risker than a classical bond as the probability to be in default is reduced. The fact that is no time lag between event of default and recovery payment should have a positive impact on the pricing of the structure. These two differences should impact negatively the yield of the structure versus classical bond yield.

We use the Monte-Carlo simulation method to compute the structure price. The Monte Carlo method principle is to sample n times a random path for the underlying in a risk neutral world and then compute n times the structure payoff. When the coupon is known, the structure possible payoff can be defined.

Table 2
Possible payoffs of the structure function of simulated level (SL) and barrier level (BL) and recovery amount (RA)
This table shows the 6 possible payoffs of the structure

	1		2		3		4		5		6	
Years	SL vs BL	Payoff	SL vs BL	Payoff	SL vs BL	Payoff	SL vs BL	Payoff	SL vs BL	Payoff	SL vs BL	Payoff
1	SL < BL	RA	SL > BL	Coupon	SL > BL	Coupon	SL > BL	Coupon	SL > BL	Coupon	SL > BL	Coupon
2		0	SL < BL	RA	SL > BL	Coupon	SL > BL	Coupon	SL > BL	Coupon	SL > BL	Coupon
3		0		0	SL < BL	RA	SL > BL	Coupon	SL > BL	Coupon	SL > BL	Coupon
4		0		0		0	SL < BL	RA	SL > BL	Coupon	SL > BL	Coupon
5		0		0		0		0	SL < BL	RA	SL > BL	RA

The price of the structure is the mean of all n payoff discounted with the risk free rate. In our pricing model, we use the classical stock behavior used by Black, Scholes and Merton (1973). If $S(T)$ be the simulated spot level at time T, we have :

$$S(T) = S(0)\,[\exp[\left(\mu - \frac{\sigma^2}{2}\right)T + \ \sigma\epsilon\sqrt{T}]]$$

As we have in our structure a decision rule for each year corresponding to the annual coupon payment date, we have a path dependent structure. Even if the structure can mature after 5 years, we need to simulate an underlying level for each year. Thus :

$$T = 1$$

The risk neutral world assumption assumes that the drift is:

$$\mu = r - d$$

Where r is the risk free rate and d is the dividend yield. σ is the implied volatility of the underlying.

$$\epsilon \sim N\ (0,1)$$

The structure has 5 years maturity but can stop each year if the underlying is below the barrier level. Then we have to use a path dependent simulation method. We simulate the underlying for each year and compute the payoff structure for each year. We first start by computing the price of the structure as we knew the coupon C. As the structure is path dependent, we simulate S(T) for T from 1 to 5 and then compute a payoff for each year of the structure. We repeat the simulation n times. In order to take into account the maturity and the volatility skew in the volatility level, we use different levels of implied volatility. We have volatility levels for T from 1 to 5 and also for different moneyness levels going from -50% to +60% of the underlying start level. We can then write the simulation as follows:

$$S(j,m,n) = S(j-1)\exp\left(r_j - d\right) - \frac{\sigma_{j,m}^2}{2} + \sigma_{j,m} * N(0,1)_{n,j}$$

Where j ranges from 1 to 5 years, m from -50% to 60% by 5%, n from 1 to 20 000.

By comparing the simulated index level and the fixed barrier level, we compute the structure payoff. There are 6 possible payoffs depending on the year when the simulated price is below the barrier level (see Table 2) If the simulated price never goes below the barrier level, the structure pays an annual coupon C and 100% at maturity.

In terms of option category, this payoff is close to a digital option. The digital option pay a fixed amount if the underlying index is above a barrier level. The fixed amount received by the purchaser of the option is invariant to the distance between the observed level and the barrier.

For each simulation, the structure's price is the sum of yearly discounted payoffs. If the first year simulated level of the underlying index is below the barrier level, the payoff is the recovery amount, the discounted payoff is computed by discounting the recovery amount with the one year interest rate. In the payoff 2, the first year coupon and the recovery amount are both discounted with the 2 years tenor interest rate. Then, for each payoff, we can use from 1 year to 5 years swap rate to discount the payoff.

When we know the coupon, we define the structure price as the sum of discounted cash flow. The aim of our study is to compute the coupon with all input parameters. In order to

compute the coupon structure, we compute the price for a continuum of coupon level going from 0% to 14% with increment of 0.35%. This is an arbitrary choice between the continuum size and the time computation as we run 20 000 simulations for each continuum points. As we have the price for the 40 possible coupons, we select the coupon corresponding to the closest price to the par (100%) as the coupon with a nominal of 100% corresponding to the yield of the structure.

3 Data

The equity index we us as underlying of the structure is the Eurostoxx50. Eurostoxx 50 represents the 50 biggest market capitalizations of the Euro Zone listed firms. The Eurostoxx50 we use is the "price" index meaning that it does not "capitalize" the dividends paid by the firm member of the index. This index offers a good liquidity on listed futures and listed option markets. The OTC market is also very active in vanilla options and on more complex structures using Bermudian or digital options.

We use daily data from 04/02/2011 to 27/01/2015. It is interesting to start the data set at this date because volatility data on deep out of the money put option is available from 2011. Before, there was no market for the very out of the money strike.

3.1 Interest rates

The € swap rate curve is used with five tenors from 1 to 5 years. The interest rate curve is used at two stage in the model. On the one hand, the swap rate is used for the drift part of the Black Scholes. On the other hand, it is used to compute the present value of expected cash flows. At this stage it seems important to mention that there is a fictive cash movement of the nominal amount at the issue date and at the maturity of the synthetic equity structure. We can also make the same synthetic structure without nominal cash amount transfer between the investor and the counterparty. Then it comes to make the structure via equity swap that is strictly the same if financial terms expect the credit spread of the counterparty. In our synthetic equity structure, the fact that there is no credit risk able to use the swap curve without any credit risk premium. The swap curve comes from Bloomberg.

Table 3
Descriptive statistics on interest rate in %
This table shows some descriptive statistics on interest € swap rate from 04/02/2001 to 27/01/2015

Maturity	Average	Minimum	Median	Maximum
1 year	0,7495	0,0955	0,4235	2,0025
2 years	0,9505	0,133	0,6973	2,79
3 years	1,1219	0,1896	0,891	3,026
4 years	1,3061	0,263	1,097	3,229
5 years	1,4807	0,341	1,301	3,3665

3.2 Dividend Yield

The Eurostoxx 50 historical dividend yield comes from Bloomberg. It reflects the average anticipated level by the markets. The dividend yield is taken into account by the market to compute Eurostoxx 50 forward levels at different tenors. In our model, it is a Black Scholes drift component.

Table 4
Descriptive statistics on dividend yield in %
This table shows some descriptive statistics on Eurostoxx 50 historical dividend yield from 04/02/2001 to 27/01/2015

Average	Minimum	Median	Maximum
4,32	3,33	4,18	6,38

3.3 Volatility

The pricing model we use to compute the structure price needs several level of volatility. The volatility matrix we use takes into account 3 dimensions: dates, volatility skew and tenors. The dates go from 04/02/2011 to 27/02/2015 on a daily basis. The volatility skew reflects the change in implied volatility level for different strike levels. The strike is defined as a % of the "at the money level". The tenors reflects the maturity on which the implied volatility is applied.

Table 5 shows the implied volatility matrix structure we use for the pricing. We use such a matrix for each day from 04/02/2011 to 27/01/2015. The volatility data comes from Citibank.

Table 5
Struture of volatility matrix for volatility
This table illustrates the structure of implied volatility function of moneynes and maturities. This table shows the volatility average level on the observed period 04/02/2001 – 27/01/2015

	Moneyness																	
Maturity	-50%	-40%	-30%	-25%	-20%	-15%	-10%	-5%	0%	5%	10%	15%	20%	25%	30%	40%	50%	60%
1 year	36,84%	32,71%	29,18%	27,59%	26,10%	24,71%	23,40%	22,18%	21,05%	20,02%	19,11%	18,32%	17,67%	17,17%	16,80%	16,46%	16,49%	16,72%
2 years	31,87%	29,05%	26,63%	25,54%	24,52%	23,57%	22,69%	21,88%	21,13%	20,44%	19,82%	19,27%	18,77%	18,34%	17,97%	17,40%	17,06%	16,90%
3 years	29,62%	27,41%	25,48%	24,62%	23,82%	23,08%	22,39%	21,75%	21,17%	20,64%	20,15%	19,71%	19,31%	18,95%	18,63%	18,09%	17,69%	17,41%
4 years	28,38%	26,50%	24,86%	24,13%	23,45%	22,82%	22,25%	21,71%	21,22%	20,77%	20,36%	19,99%	19,65%	19,34%	19,06%	18,57%	18,18%	17,88%
5 years	27,62%	25,96%	24,50%	23,85%	23,25%	22,70%	22,19%	21,72%	21,29%	20,90%	20,54%	20,21%	19,92%	19,64%	19,39%	18,96%	18,60%	18,31%

3.4 Corporates bond spread

We use the corporates bond spread versus interest swap rate (IRS). These spreads are extracted from Bloomberg and come from Merrill Lynch indexes. We have these spread for 5 rating classes from AAA to BB. These spread are computed from mid-price of the bond.

Table 6
Descriptive statistics of corporate credit spread
This table show descriptive statistcs on corporate bond credit spread versus IRS as provided by Bloomberg (source Merrill Lynch).

	AAA	AA	A	BBB	BB
Mean	0,429	0,644	1,082	2,076	3,970
Minimum	0,140	0,180	0,390	0,950	2,160
Maximum	0,820	1,850	2,850	4,020	7,060
Median	0,450	0,490	0,820	1,890	3,600

All 4 statistics indicate a positive link between rating and spread. The spread is higher for lower rating which reflects a normal situation where the risk is remunerated by a higher potential return reflected by the credit spread. At this stage, it seems important to deal with the problematic of duration. The spread reflects, for all 5 rating classes, the difference between the yield and the IRS with the same duration. In order to compare the spread of the different rating classes, we assume that the duration is the same. Table 1 shows the average daily duration of each 5 rating class are close.

4 Results

4.1 Synthetic equity structure average coupon

Table 7
Synthetic equity structure average coupon
This table show the average coupon for different recovery rates and barrier levels.

Recovery Rate	Barrier level								
	30%	35%	40%	45%	50%	55%	60%	65%	70%
20%	2,256	2,697	3,220	3,879	4,744	5,797	7,086	8,706	10,634
30%	2,157	2,544	3,002	3,588	4,342	5,272	6,405	7,812	9,573
40%	2,044	2,391	2,791	3,299	3,943	4,747	5,706	6,926	8,433
50%	1,947	2,257	2,574	2,994	3,535	4,202	5,023	6,028	7,289

We see from table 7 two kind of relationship between coupon level, barrier level and recovery rate. Firstly the positive relationship between coupon and barrier level for a given recovery rate. A higher barrier level increases the probability to get the recovery rate instead of 100% of the nominal value and the coupon. This relationship shows that the synthetic equity structure remunerates the risk to get the recovery rate by a higher coupon level. Secondly the negative relationship between the coupon level and the recovery rate for a given barrier level. This relation reflects the fact the coupon is higher for an equity structure in which the loss given default is higher. Then again, the risk in the synthetic equity structure is remunerated.

4.2 Synthetic equity structure spread vs corporate bond yield spread

This section compares the synthetic equity spread and the corporate bond yield spread, in order to see in what extend one of both instrument dominate the other one in terms of yield.

We compare in this section 2 spreads. The spread on corporate bond yield is a spread versus IRS with duration adjustment (source Bloomberg). The spread we compute for the synthetic equity structure is a spread versus 5 years tenor IRS.

The comparison with the AAA rating class is not relevant as the number of issuer belonging to this class is very low.

Table 8 to 11 provide statistics on the spread (difference between the yield and swap rate) difference between the synthetic equity structure and the corporate bond yields for rating classes AA, A, BBB and BB. These statistics are computed from the 961 data points from 04/02/2011 and 27/02/2015. For the average, * means the average is significantly different from 0 at 95% confidence level.

4.2.1 AA rating

Table 8
Descriptive statistics on spread for AA rating class

This table show the descriptive statistics on the spread difference (in%) between synthetic equity structure and AA corporate bonds yield

Recovery rate	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	0,132*	0,573*	1,095*	1,754*	2,619*	3,672*	4,962*	6,582*	8,509*
Minimum	-0,420	-0,240	0,128	0,461	1,111	1,861	2,861	4,100	5,724
Maximum	1,217	2,222	3,272	4,322	5,722	7,122	8,872	10,354	11,873
Median	0,100	0,466	0,891	1,522	2,430	3,319	4,633	6,276	8,394
% of difference >0	58,22%	95,81%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	0,033*	0,420*	0,877*	1,464*	2,218*	3,148*	4,281*	5,688*	7,448*
Minimum	-1,134	-0,290	0,060	0,411	0,811	1,511	2,511	3,561	5,023
Maximum	1,016	1,716	2,572	3,622	4,717	6,072	7,472	9,222	10,816
Median	-0,014	0,327	0,746	1,281	2,066	2,883	3,966	5,363	7,313
% of difference >0	48,23%	90,87%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-0,081*	0,266*	0,667*	1,175*	1,818*	2,622*	3,581*	4,802*	6,309*
Minimum	-1,211	-0,342	0,008	0,273	0,761	1,287	2,161	2,911	4,269
Maximum	0,745	1,366	2,066	2,922	3,972	5,067	6,154	7,822	9,222
Median	-0,091	0,234	0,562	1,045	1,708	2,437	3,341	4,589	6,172
% of difference >0	32,98%	79,91%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-0,177*	0,132*	0,45*	0,870*	1,411*	2,077*	2,898*	3,903*	5,165*
Minimum	-1,211	-0,419	-0,145	0,111	0,461	1,111	1,559	2,511	3,561
Maximum	0,395	1,023	1,679	2,222	2,967	4,017	5,022	6,266	7,629
Median	-0,160	0,125	0,382	0,805	1,322	1,931	2,769	3,732	5,059
% of difference >0	20,52%	65,74%	96,46%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%

4.2.2 A rating

Table 9
Descriptive statistics on spread for A rating class

This table show the descriptive statistics on the spread difference (in%) between synthetic equity structure and A corporate bonds yield

Recovery rate	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-0,307*	0,134*	0,656*	1,315*	2,180*	3,233*	4,523*	6,142*	8,071*
Minimum	-1,163	-0,480	-0,113	0,221	0,869	1,621	2,619	3,840	5,484
Maximum	0,589	1,414	2,464	3,514	5,043	6,539	8,064	9,814	10,893
Median	-0,321	0,098	0,551	1,202	2,116	3,016	4,323	5,945	8,083
% of difference >0	10,20%	65,95%	97,74%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-0,406*	-0,019*	0,439*	1,025*	1,779*	2,709*	3,842*	5,249*	7,009*
Minimum	-1,388	-0,763	-0,182	0,169	0,571	1,271	2,269	3,319	4,783
Maximum	0,275	1,064	1,989	2,814	4,089	5,489	6,889	8,639	9,839
Median	-0,392	-0,038	0,393	0,947	1,736	2,557	3,657	5,046	6,986
% of difference >0	3,01%	44,15%	92,27%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	0,520*	0,0173*	0,228*	0,736*	1,380*	2,183*	3,142*	4,363*	5,870*
Minimum	-1,513	-0,984	-0,413	0,013	0,519	1,037	1,919	2,671	4,019
Maximum	0,180	0,589	1,289	2,114	3,164	4,439	5,614	7,014	8,639
Median	-0,464	-0,149	0,196	0,674	1,325	2,111	3,036	4,275	5,860
% of difference >0	0,43%	22,77%	79,59%	100,00%	100,00%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	0,0616*	0,306*	0,011	0,430*	0,972*	1,639*	2,459	3,465*	4,726*
Minimum	-1,774	-1,214	-0,763	-0,200	0,221	0,869	1,329	2,269	3,319
Maximum	-0,042	0,299	0,739	1,414	2,339	3,164	4,214	5,614	6,889
Median	-0,542	-0,291	0,004	0,400	0,936	1,579	2,386	3,412	4,733
% of difference >0	0,00%	8,27%	50,70%	94,52%	100,00%	100,00%	100,00%	100,00%	100,00%

4.2.3 BBB rating

Table 10
Descriptive statistics on spread for BBB rating class

This table show the descriptive statistics on the spread difference (in%) between synthetic equity structure and BBB corporate bonds yield

Recovery rate	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-1,300*	-0,859*	-0,336*	0,322*	1,187*	2,240*	3,530*	5,150*	7,078*
Minimum	-2,611	-1,958	-1,211	-0,638	0,063	1,058	2,079	3,310	4,924
Maximum	-0,286	0,192	1,242	2,292	3,753	5,229	6,842	8,494	9,289
Median	-1,233	-0,846	-0,353	0,231	1,091	2,078	3,351	4,964	7,067
% of difference >0	0,00%	0,97%	13,96%	85,28%	100,00%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-1,399*	-1,012	-0,554*	0,032*	0,786*	1,716*	2,849*	4,256*	6,016*
Minimum	-2,909	-2,261	-1,561	-0,889	-0,189	0,721	1,725	2,779	4,224
Maximum	-0,286	-0,121	0,679	1,592	2,779	4,179	5,579	7,329	8,494
Median	-1,316	-0,949	-0,569	0,005	0,726	1,601	2,700	4,096	5,959
% of difference >0	0,00%	0,00%	4,83%	51,24%	99,79%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-1,153*	-1,166*	-0,765*	-0,257*	0,387*	1,190*	2,149*	3,370*	4,877*
Minimum	-2,961	-2,577	-1,911	-1,211	-0,511	0,300	1,113	2,121	3,479
Maximum	-0,471	-0,241	0,109	0,892	1,942	3,129	4,294	5,792	7,329
Median	-1,394	-1,094	-0,713	-0,259	0,358	1,111	2,036	3,270	4,818
% of difference >0	0,00%	0,00%	1,18%	16,97%	92,48%	100,00%	100,00%	100,00%	100,00%
Recovery rate	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-1,609*	-1,300*	-0,982*	-0,562*	-0,021*	0,646*	1,466*	2,472*	3,733*
Minimum	-3,259	-2,752	-2,261	-1,800	-1,159	-0,400	0,650	1,463	2,779
Maximum	-0,591	-0,241	0,065	0,415	1,029	1,942	2,992	4,294	5,579
Median	-1,525	-1,255	-0,937	-0,556	0,001	0,604	1,409	2,407	3,693
% of difference >0	0,00%	0,00%	0,32%	3,97%	50,48%	98,71%	100,00%	100,00%	100,00%

4.2.4 BB rating

Table 11
Descriptive statistics on spread for BB rating class

This table show the descriptive statistics on the spread difference (in%) between synthetic equity structure and BB corporate bonds yield

Recovery rate	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**	**20%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-3,194*	-2,753*	-2,231*	-1,572*	-0,707*	0,346*	1,636*	3,255*	5,183*
Minimum	-5,231	-4,608	-3,831	-2,879	-1,941	-0,812	0,239	1,653	3,764
Maximum	-1,611	-1,263	-0,771	-0,191	0,838	2,169	3,754	5,504	7,160
Median	-3,054	-2,720	-2,265	-1,574	-0,702	0,267	1,523	3,163	5,144
% of difference >0	0,00%	0,00%	0,00%	0,00%	6,55%	76,37%	100,00%	100,00%	100,00%
Recovery rate	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**	**30%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-3,294*	-2,907*	-2,449*	-1,863*	-1,109*	-0,179*	0,954*	2,361*	4,122*
Minimum	-5,562	-4,881	-4,181	-3,462	-2,431	-1,381	-0,145	0,953	3,039
Maximum	-1,613	-1,459	-0,913	-0,421	0,279	1,210	2,519	4,269	5,814
Median	-3,138	-2,807	-2,407	-1,885	-1,095	-0,197	0,866	2,266	4,019
% of difference >0	0,00%	0,00%	0,00%	0,00%	2,26%	36,63%	99,46%	100,00%	100,00%
Recovery rate	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**	**40%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-3,407*	-3,060*	-2,660*	-2,152*	-1,508*	0,704*	0,255*	1,475*	2,983*
Minimum	-5,581	-5,212	-4,531	-3,831	-3,121	-2,081	-0,891	0,253	1,989
Maximum	-1,809	-1,459	-1,121	-0,563	-0,071	0,838	1,806	3,030	4,710
Median	-3,225	-2,973	-2,624	-2,104	-1,498	-0,676	0,251	1,439	2,912
% of difference >0	0,00%	0,00%	0,00%	0,00%	0,00%	8,16%	72,07%	100,00%	100,00%
Recovery rate	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**	**50%**
Barrier Level	**30%**	**35%**	**40%**	**45%**	**50%**	**55%**	**60%**	**65%**	**70%**
Average	-3,503*	-3,194*	-2,876*	-2,457*	-1,916*	-1,249*	-0,428*	0,577*	1,838*
Minimum	-5,912	-5,562	-4,881	-4,258	-3,812	-2,858	-1,731	-0,680	0,642
Maximum	-1,809	-1,471	-1,263	-0,911	-0,421	0,279	1,188	2,260	3,638
Median	-3,289	-3,053	-2,771	-2,374	-1,842	-1,254	-0,434	0,554	1,824
% of difference >0	0,00%	0,00%	0,00%	0,00%	0,00%	1,93%	22,56%	89,04%	100,00%

Depending on the 3 variables rating, recovery rates and barrier level, we see on these statistics that the synthetic equity structure yield partially dominate the corporate bond yield.

For the AA rating (table 8) and both recovery 20% and 30%, the difference spread is significantly positive for all barrier levels. The difference is positive in 100% cases for the recovery rates and all barrier levels from 40% to 70%.We can conclude that the synthetic equity structure strictly dominate the corporate bond in terms of yield for these two recovery rate levels. For recovery rates 40% and 50%, the domination is still valid for all barrier level except 30%.

For the A rating (table 9), the dominance is valid for all recovery rate and barrier from 45% to 70%.

For the BBB rating (table 10), the synthetic equity structure offers better yield than corporate bonds for barrier level 50% to 70% and with lower recovery rate 20% and 30%.

The BB rating (table 11) offers better yield than the synthetic equity structure for almost all recovery rate and barrier level except very high barrier level.

With the 40% recovery rate that is usually taken as an average recovery rate on Credit Default Swap pricing, we see the synthetic equity structure offers a better yield than AA rating class for all barrier level except the first two lowest barrier level 30% and 35%. For all barrier levels going from 40% to 70% , in 100% cases the equity structure offers a better yield than AA corporate bonds with an average difference going from 0.667% (barrier level 40%) to 6.309% (barrier level 70%). The average spread increase with the barrier level which reflects that the synthetic equity structure remunerate for the risk to be in failure. The probability that the structure be in failure (equivalent to probability of default in the credit world) increases with the barrier level so the increasing yield reflect this risk taken by the investor. In the pricing model, the barrier level will condition the probability to get the recovery rate instead of 100%. Then the average payoff decrease as the barrier level increase. In order to make the average payoff going to the nominal value, ie 100%, the coupon needs to increase as compensation. We are in a "normal" risk-return situation where the risk is remunerated by a better yield. With a similar logic, we see also on table 8 the probability that the synthetic equity structure offers a better spread than AA rating class increase with lower recovery rate. The probability is still 100% for each barrier level from 40% to 70% for recovery 20% and 30%. For the two barrier level 30% and 35% for which the probability is not 100%, this probability increase with the recovery rate. With a 35% barrier level, the probability to observe a better spread is 79.91% for the 40% recovery and respectively 90.87% and 95.81% for recovery 30% and 35%. On aggregate, the % spread positive between synthetic equity structure and corporate bond AA rating is 91.35% (94.89% for recovery 20%, 93.23% for recovery 30%, 90.32% for recovery 40% and 86.97% for recovery 50%). This aggregate % of spread positive goes to 79.71% for rating AA, 58.8% for rating BBB and 33.75% for rating BB. On the 9 barrier levels used (from 30% to 70% by 5%), it is interesting to measure the number for which 100% datapoints give a better synthetic equity structure than corporate bond yield . It is respectively 7 on 9 for rating AA coupon

The next section will provide elements to see if the better spread offered by the equity structure reflects a higher risk of failure compared to the probability of default of corporate bonds.

4.3 Probability of default

Results in terms of yield presented above able to conclude that the equity derivative structure offers higher yield than corporate bonds portfolio for some rating classes, barrier levels and recovery rates. We demonstrate in this section that the equivalent probability of default compared to observed probability of default for corporate bonds can justify a positive difference but not in all cases.

For each barrier level, we compute an historical structure failure probability. We use Eurostoxx 50 price index daily return to build 1 to 5 years rolling returns in order to compute structure failure probability for each year (Table 12). Dataset goes from 1987 to 2015.

Table 12
Probability of equity structure to be in default equivalent
This table show the probability the equity structure to be in failure or default equivalent

	30%	35%	40%	45%	50%	55%	60%	65%	70%
1 year	0,000%	0,000%	0,000%	0,000%	0,028%	0,641%	2,023%	3,647%	5,371%
2 years	0,000%	0,000%	0,000%	0,103%	1,551%	4,831%	6,382%	8,273%	10,223%
3 years	0,000%	0,015%	0,184%	1,273%	2,608%	4,264%	6,289%	9,357%	14,481%
4 years	0,000%	0,000%	0,000%	0,016%	0,622%	3,493%	6,476%	10,911%	17,020%
5 years	0,000%	0,000%	0,000%	0,000%	0,399%	2,625%	7,161%	9,370%	13,042%

This table should be read as followed: the equity structure with 35% barrier level has a probability of 0% to be in failure in all 5 years except in year 3 with a failure probability of 0.01%. From this probability, we build the equity structure failure cumulative probability. This cumulative probability is computed by multiplying the survival probability (1-failure probability) for each year and then make 1- product of each survival probability.

Table 13
Equity structure cumulative probability to be in default equivalent
This table shows the cumulative probability the equity structure to be in failure or default equivalent.

Barrier level	30%	35%	40%	45%	50%	55%	60%	65%	70%
Cumulative probability of failure	0,000%	0,015%	0,184%	1,391%	5,122%	14,929%	25,368%	35,317%	47,576%

In order to make a risk / yield profile comparison between the corporate bond index and the equity structure, we compare the cumulative probability of equity structure failure and the 5 years probability of default .We can compare these figures to the 5 years probability of default given by Standard & Poors.

Table 14
Cumulative probability of default for corporate bonds
This table shows the 5 years S&P probability of default.

Rating	Probability of default
AA	0,36%
A	0,62%
BBB	2,15%
BB	8,35%

4.4 Risk Yield profile comparisons

A risk yield analysis is done by detecting the case on which we can see an anomaly, meaning a case where the synthetic equity structure offers a better spread whereas its risk is lower in terms of probability of default.

We define as anomaly a situation where the average spread between the synthetic equity structure and the corporate bond is significantly positive and the difference on probability of default is negative. In these case, we can stipulate that the synthetic equity structure offers a better risk yield profile than the corporate bonds. Table 15 gives the results for the 4 rating class. We see in the table 15 that the synthetic equity structure offers a better risk yield profile than the AA rating corporate bonds for barrier level 35% and 40%% and for all recovery rates. For barrier level 30%, this is also the case for recovery rates 20% and 30%. For the rating AA, we have 8 cases of anomaly on 36. For the rating A, the number of anomaly decrease to 4 on 36, also for barrier level 35% and 40%. For BBB rating, the only 2 cases of anomaly are seen for barrier level 45% and the 2 lowest recovery 20% and 30%. The BB rating class systematically offers a better risk return profile than the synthetic equity structure.

Table 15
Test of anomaly in Risk Return profile
This table shows for all rating class the situation where the average spread is significantly positive and the difference between equity structure probability of failure and corporate bond probability of default is negative.

Rating	Recovery Rate	Barrier Level								
		30%	35%	40%	45%	50%	55%	60%	65%	70%
AA	20%	**YES**	**YES**	**YES**	NO	NO	NO	NO	NO	NO
	30%	**YES**	**YES**	**YES**	NO	NO	NO	NO	NO	NO
	40%	NO	**YES**	**YES**	NO	NO	NO	NO	NO	NO
	50%	NO	**YES**	**YES**	NO	NO	NO	NO	NO	NO
BBB	20%	NO	**YES**	**YES**	NO	NO	NO	NO	NO	NO
	30%	NO	NO	**YES**	NO	NO	NO	NO	NO	NO
	40%	NO	NO	**YES**	NO	NO	NO	NO	NO	NO
	50%	NO	NO	NO	NO	NO	NO	NO	NO	NO
BB	20%	NO	NO	NO	**YES**	NO	NO	NO	NO	NO
	30%	NO	NO	NO	**YES**	NO	NO	NO	NO	NO
	40%	NO	NO	NO	NO	NO	NO	NO	NO	NO
	50%	NO	NO	NO	NO	NO	NO	NO	NO	NO
B	20%	NO	NO	NO	NO	NO	NO	NO	NO	NO
	30%	NO	NO	NO	NO	NO	NO	NO	NO	NO
	40%	NO	NO	NO	NO	NO	NO	NO	NO	NO
	50%	NO	NO	NO	NO	NO	NO	NO	NO	NO

This table shows several things. Firstly there are ways to improve the risk yield profile of corporate bond portfolio by "replicating" a corporate bond portfolio with a synthetic equity derivatives products. Secondly, the level of coupon of these equity structure can provide information on the "implied" level of recovery rate on the corporate bonds market.

5 Conclusion

This paper proposes an empirical study aiming to compare two kinds of financial assets, corporate bond index and a synthetic equity derivative structure that mimic corporate bonds indexes by replacing the credit spread by an equity option short position. In a buy and hold framework, we show that the use of equity derivatives can improve the risk yield profile of a corporate bond portfolio.

Even if we show that a corporate bond investor can improve its investment profile by replacing high rated corporate bond by such an equity derivative structure, we only take in this empirical study a buy and hold view as we don't compute any return of both asset classes. Thus, we cannot infer a classical risk return study that compares average return, volatility

and then compares Sharpe ratio. This is possible extension of this paper to compare both return profile with mark to market framework.

Another possible extension is to analyze deeper the link between the credit spread and the valuation of a digital put option on equity. Theoretically the credit spread is linked with probability of default which is defined as the probability that assets of a firm is below its liabilities. Then the probability of default is computed with the volatility of assets. An open question is the possible extension of the implied volatility that is used to price out of the money put option on equity and the volatility of asset used to compute the probability of default.

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