Liquidity premium in CDS markets

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Liquidity Risk,
Outline

1. Motivation
2. Model
3. Data
4. Calibration results
5. Conclusion
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Motivation 1

Previous literature on liquidity in credit markets

- **No liquidity premium in CDS markets**
  Longstaff, Mithal, and Neis (2005)

- **Liquidity risk in CDS markets, but no estimation of the liquidity premium**
  Tang and Yan (2007)
  Bongaerts, de Jong, and Driessen (2011)

- **Liquidity premium dynamics in reduced-form models: uncorrelated risk factors**
  Chen, Cheng, and Wu (2008)
  Chen, Fabozzi, and Sverdlove (2010)

- **Liquidity premium dynamics in reduced-form models: correlated risk factors**
  Bühler and Trapp (2010)
Motivation 2

PANEL A: Mid CDS premium ($S^{mid}$)

PANEL B: Bid-ask spread ($BA$)

PANEL C: Bid-ask spread relative to mid quote ($BA^{mid}_{rel}$)
Objectives and Contributions

- CDS quotes contain liquidity premia
- Strong interlinkage of liquidity and default risk premium
- Allocation of liquidity premium between protection buyers and sellers
- Simple and robust framework
- Investigation of liquidity premium before the financial crisis and in different phases of the financial crisis
- Financial and non-financial institutions
Empirical findings

• Time-varying credit and liquidity risk premia

<table>
<thead>
<tr>
<th></th>
<th>credit</th>
<th>bid-ask</th>
<th>ask liquidity ratio in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-crisis Phase</td>
<td>34.37</td>
<td>3.56</td>
<td>25.70</td>
</tr>
<tr>
<td>Subprime Phase</td>
<td>70.12</td>
<td>5.21</td>
<td>23.5</td>
</tr>
<tr>
<td>Systemic Phase</td>
<td>178.68</td>
<td>14.83</td>
<td>41.70</td>
</tr>
<tr>
<td>Recovery Phase</td>
<td>103.62</td>
<td>7.85</td>
<td>32.10</td>
</tr>
<tr>
<td>Sovereign Phase</td>
<td>101.30</td>
<td>5.95</td>
<td>43.8</td>
</tr>
</tbody>
</table>

• The larger the credit premium the larger the liquidity premium of protection seller and protection buyer

• Liquidity premium allocated to protection buyers

• Financial institutions on average have a lower credit risk premium than non-financials before the sovereign phase

• Financial institutions on average have a lower liquidity premium than non-financials
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Economy

- **Credit or default premium** $S_t^{def}$
  \[
  s_t^{def} = \ln(S_t^{def})
  
  s_{t+1} = s_t + \eta_{t+1}
  \]

- **Ask CDS price** $S_t^{ask}$
  \[
  s_t^{ask} = \ln(S_t^{ask}) = s_t^{def} + r_t ba_t
  \]

- **Bid CDS price** $S_t^{bid}$
  \[
  s_t^{bid} = \ln(S_t^{bid}) = s_t^{def} - (1 - r_t) ba_t
  \]

- **Bid-ask spread of logarithmized prices**
  \[
  ba_t^{def} = s_t^{ask} - s_t^{bid}
  \]

- **liquidity state variable**
  \[
  r_t = \alpha + \beta r_{t-1} + \sqrt{(1 - r_{t-1}) r_{t-1}} \epsilon_t, \quad \text{with} \quad 0 \leq \alpha \leq 1, \ -1 \leq \beta \leq 1
  \]
Estimation Equations

- **Observation equation**: changes in log-ask premium

\[
y_t = s_{t}^{ask} - s_{t-1}^{ask} \\
= r_t \Delta_{ba} - r_{t-1} \Delta_{ba_{t-1}} + \eta_t \\
= \begin{bmatrix} \Delta_{ba} & -\Delta_{ba_{t-1}} \end{bmatrix} \begin{bmatrix} r_t \\ r_{t-1} \end{bmatrix} + \eta_t \\
= H_t x_t + \eta_t
\]

- **Transition equation**: liquidity state variables

\[
r_t = \alpha + \beta r_{t-1} + \sqrt{(1 - r_{t-1})} r_{t-1} \epsilon_t \\
x_t = A + F x_{t-1} + \sqrt{(1 - r_{t-1})} r_{t-1} w_t
\]

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State space model

- **Linear dynamic system**

\[
\begin{align*}
y_t &= H_t x_t + \eta_t, \\
x_t &= A + F x_{t-1} + L_{t-1} w_t \\
\eta_t &\sim N(0, \sigma_\eta^2) \\
w_t &\sim N(0, Q) \\
Q &= \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & 0 \end{bmatrix} \\
M &= \text{Cov} [w_t, \eta_t] = \begin{bmatrix} \rho_\epsilon, \eta \sigma_\epsilon \sigma_\eta & 0 \\ \rho_\epsilon, \eta \sigma_\epsilon \sigma_\eta & 0 \end{bmatrix}
\end{align*}
\]

- **State variable** \(x_t\) is estimated by means of Kalman filter
Parameter Estimation

- **Quasi maximum likelihood**

\[ \hat{\Theta} = \arg \max \ln \left( f \left( y_1, \ldots, y_T \right) \right) = -\frac{1}{2} \sum_{t=1}^{T} \left( \ln \left( V_{t+1} \right) + \frac{\left( y_{t+1} - y_{t+1|t} \right)^2}{V_{t+1}} \right) \]

- **Global optimization procedure**

1. \( N = 200 \) initial parameter sets \( \Theta_0^1, \ldots, \Theta_0^N \)
2. For each initial parameter set \( \Theta_0^k, \quad k = 1, \ldots, N \), the Kalman filter generates a time series of \( y_{t+1|t} \) and \( x_{t|t} \)
3. For each \( k = 1, \ldots, N \), search for \( \hat{\Theta}^k \) that maximize the objective function, given \( y_t, y_{t|t-1}, x_{t|t} \quad t = 1, \ldots, T \)
4. Set \( \hat{\Theta} \) equal to \( \hat{\Theta}^k \) with the maximal log-likelihood function
5. Reinitialize the procedure by setting \( \Theta_0^1 = \hat{\Theta} \) and randomly generate \( N - 1 \) parameter sets \( \Theta_0^2, \ldots, \Theta_0^N \) and return to the second step
6. As soon as the parameter estimates \( \hat{\Theta} \) converge, stop the procedure
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Data

- 5 year CDS bid and ask prices
- ranging from January 2004 to September 2010
- 118 names from iTraxx Europe
  - automobile and industry 29
  - consumer and service 26
  - financial 23
  - energy 20
  - telecommunications 20
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Default and liquidity premia

**PANEL A: Model-implied CDS premium (S_{\text{def}})**

**PANEL B: Ask liquidity premium (S_{L,\text{ask}})**

**PANEL C: Bid liquidity premium (S_{L,\text{bid}})**
Ask liquidity ratio
Histogram of ask liquidity ratio

Ask liquidity proportion ($R$)

- Financial
  - Pre-crisis
- Subprime
- Systemic
- Recovery
- Non-Financial
  - Pre-crisis
- Subprime
- Systemic
- Recovery
- Sovereign

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Liquidity premium in CDS markets
Deviation of estimated credit premium from mid-price

Pre-crisis  Subprime  Systemic  Recovery  Sovereign
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Conclusion

- **CDS default premium**
  - Mid-prices underestimate the CDS default risk
  - The default premium increases during the crisis and reaches a maximum in the systemic period
  - Financial institutions have a lower default premium than non-financials
  - Financials have comparatively low default premium during the systemic phase and comparatively large default premium during the sovereign crisis period

- **Liquidity premium**
  - The liquidity premium is mostly allocated to protection buyers
  - The ask liquidity ratio increases significantly during periods of large credit premia
  - Financials exhibit generally lower ask liquidity ratios