Market Structure, Counterparty Risk, and Systemic Risk

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Counterparty Risk

- **Counterparty**: other side of ongoing financial agreement.
  - A bank enters into a swap with you on the S&P 500.

- **Counterparty Risk**
  - Risk resulting from default/bankruptcy of a counterparty.
  - Strictly: Risk to you from one of your counterparties.
  - Broadly: Includes effects on overall market (our concern).

- This broad definition we refere to as *systemic risk*. 
Counterparty Risk to Systemic Risk

- Counterparty risk affects market when large failure looms:
  - Near-bankruptcy of Bear Stearns (May 2008)
  - Bankruptcy of Lehman Brothers (Sep 2008)
  - Bankruptcy of Refco Inc? (Oct 2005, owned #1 CME broker)

- Outstanding notional at CME before ceasing trading:
  - Bear
  - Lehman
  - Refco LLC
  - $761 BB
  - $1,150 BB
  - $130 BB

- N.B. No defaults or trade halts at CME for these events.

- Is counterparty risk an “accelerant” in financial crises?
Distress increases volatility sharply and significantly.
- Widens spreads: transactions costs $\uparrow$; market liquidity $\downarrow$.
- Volatility is pushed onto the survivors (externality).

Crisis bankruptcies have real costs:
- Virtuous, vicious circles of market and funding liquidity$^2$.
- Reduced funding liquidity affects non-financial firms also.
- Less invested in risky assets; allocative inefficiency?
- Higher unemployment: harder job searches, lower tax revenue.
- Bernanke (1983): affects credit markets; possible depression.

Market structure affects contagion and exposure to defaults.

Specifically: complete networks magnify systemic risk.
  - Difference due to differing creation of complete networks.
  - Also: financial, banking networks differ (cf Acemoglu).

Market fragility estimable with a few metrics of market core.
Can price distress volatility of differing structures.
Model: Market Structures

- Investigate two extremes of $n$-counterparty networks.

![Network Diagram]

Star network
(Market with CCP$^3$)

Complete network
(Bilateral “OTC” market)

- Each node is a counterparty (capital $K$, risk aversion $\lambda$).
- Each edge is a contract$^4$ linking counterparties $i$ and $j$.
- Contract exposure: $q_{ij} = -q_{ji}$; $q_{i<j} \overset{iid}{\sim} N(0, \eta^2)$
- Counterparty $i$’s net exposure: $Q_i = \sum_{j \neq i} q_{ij}$.
- Same net exposures ($Q_i$’s) in both networks.

$^3$Central counterparty.
$^4$A swap or forward on a risky asset.
Model: Event Timing

To study counterparty risk, events occur at discrete times.

\( t = 0 \): Bankruptcy of counterparty \( n \) occurs.
  - All contracts with counterparty \( n \) are invalidated.
  - Pushes unwanted exposure onto other \( n - 1 \) counterparties.

\( t = 1 \): Living counterparties trade in response to bankruptcy.

\( t = 2 \): Living counterparties close out bankruptcy-induced exposure.

Order of trading in a period is random, not strategic.
Model: Price Impact of Trading

- Each counterparty $i$ trades $x_i$ shares at time $t = 1$.
  - Impact has linear permanent component\(^5\).
  - Permanent component impacts prices for later traders.
- Trade ordering, price impact create low and high prices.
- Time periods are very short; two simplifying assumptions:
  1. Prices have no drift other than price impact due to trading.
  2. Price diffusion is Gaussian (not log-normal).
- Defer handling crisis-related adverse selection.

\(^5\)Price impact could arise from inventory risk cost, non-crisis adverse selection.
Suppose counterparty A is net long the market.
⇒ Other counterparties are net short the market.
These are their preferred equilibrium positions.
Thus when counterparty A defaults:
- Survivors must re-create exposure from counterparty A.
- Survivors become net sellers.

CCP market: only CCP trades; net sell.

OTC market: some counterparties will sell, some will buy.

However, counterparties trade in own interest.
- Do they rehedge immediately? Push market further?
Consider bankruptcy of a large financial firm.
Assume large market move \( r_0 \) at \( t = 0 \) induces bankruptcy.
Net exposure \( Q_n \) probably large; estimate via EVT\(^6\).

\[
\hat{Q}_n = \frac{-K}{r_0} + \frac{\eta \sqrt{n-1}}{c_n(1 - e^{-c_n \kappa_1 - d_n})} \sum_{k=1}^{\infty} \frac{(-1)^{k+1} e^{-k (c_n \kappa_1 + d_n)}}{k k!}
\]

where \( \kappa_1 = \frac{-K}{r_0 \eta \sqrt{n-1}} \) (minimum exposure causing death),
\( c_n = \frac{1}{\sqrt{2 \log(n)}} \), and \( d_n = \sqrt{2 \log(n)} - \frac{\log \log(n) + \log(16 \tan^{-1}(1))}{2 \sqrt{2 \log(n)}} \).

\(^6\)Equivalent: endow all counterparties with perfect information, examine most likely \( Q_n \mid r_0 \).
For large $Q_n$, trading at $t = 1, 2$ will move market a lot.

Move will be further in direction that caused bankruptcy.

This raises two distressing possibilities:
- Contagion: move may cause other counterparties to fail; or,
- Checkmate: hedging may bankrupt the hedger.

Counterparties anticipate these, respond selfishly.

For bilateral OTC market, all counterparties may trade.
- All hedge anticipated follow-on bankruptcy exposure $\hat{Q}_f$.
- Trouble: $\nu > 1$ (overtrading at $t = 1$) to be expected.
- Longs, shorts may largely self-segregate rehedge timing.

Thus network structure matters.
CCP anticipates follow-on bankruptcies; equilibrium yields

Follow-on bankruptcy exposure $\hat{Q}_f$ (distress exposure):

$$\hat{Q}_f = (n - 1)^{3/2} \eta \frac{\phi(\kappa_2) - \phi(\kappa_1)}{\Phi(\kappa_1)}$$

where

$$\kappa_2 = \frac{-Kp_0 / [\eta \sqrt{n - 1}]}{p_0 r_0 - \pi (\hat{Q}_n + \hat{Q}_f)} = \text{min exposure for follow-on death.}$$

Follow-on bankruptcies $\hat{b}$ (distress pervasiveness):

$$\hat{b} = (n - 1) \frac{\int_{\kappa_2}^{\kappa_1} \phi(z)dz}{\int_{-\infty}^{\kappa_1} \phi(z)dz} = (n - 1) \left(1 - \frac{\Phi(\kappa_2)}{\Phi(\kappa_1)}\right)$$
Large Bankruptcy: Equilibrium OTC Net Trade

- OTC traders anticipate one another, follow-on bankruptcies.
- However: those most at-risk rehedge quickly, others delay.
- Random trade sequence ⇒ uncertain low of rehedging $S_{n-1}$.
- Use these to solve for equilibrium OTC net trade.

\[ \kappa_2 = \frac{-Kp_0}{\eta \sqrt{n-1}(p_0r_0 + \pi E(S_{n-1}|\nu))}, \]  

\[ \hat{Q}_f = (n-1)^{3/2} \eta \frac{\phi(\kappa_2) - \phi(\kappa_1)}{\Phi(\kappa_1)}. \]

- Important to note that $\nu \geq 1$ (in $E(S_{n-1})$).
- Finding $\nu$ is hard: $n$-player (random) game; usually $c1.75$. 
Strategic Trading: All Together Now?

Proposition (Pooling)

*In bilateral OTC markets, buyers and seller may split their trades between periods 1 and 2 according to cost minimization. This pooling of buying and selling is a Bayesian Nash equilibrium.*

Proposition (Separating)

*In bilateral OTC markets, buyers and sellers may separate with buyers in one period and sellers in the other period. This separating of trade timing is a Bayesian Nash equilibrium.*

In progress: Proofs of pooling vs separating decision, effects.
Bad Behavior? Checkmate and Hunting

Proposition (Checkmate)

A large enough initial bankruptcy may yield a follow-on bankruptcy in expectation — despite any finite effort by the troubled counterparty.

Proposition (Hunting)

For a complete network of 3 or more counterparties and a large enough initial bankruptcy, two or more other counterparties may profit by driving a survivor into (follow-on) bankruptcy.
The Separating Equilibrium

- I mentioned an (extreme) possibility in bilateral OTC markets:
  - Buyers and sellers may separate when they trade.
  - Those who are same side as net rehedge rush to hedge first.
  - Those on other side wait to allow maximum distress.
  - If net rehedge makes sellers panic, net sale in period 1 is:

\[
- E(\sum_{i=1}^{n-1} [x_i] - |\sum_{i=1}^{n-1} x_i| = -\hat{Q}_n - \hat{Q}_f) 
\approx -(n-1)^{3/2} \eta \phi(\mu^*) - (\hat{Q}_n + \hat{Q}_f)(1 - \Phi(\mu^*)) \tag{7}
\]

where \( \mu^* = \frac{\hat{Q}_n + \hat{Q}_f}{(n-1)^{3/2} \eta} \) (net rehedge in std devs/survivor) and \( \phi, \Phi \) are standard normal pdf, cdf.
Large Bankruptcies: Indicative Distress

- Consider large bankruptcy for \( n = 10 \) counterparties\(^7\).
- Std deviation of bilateral contract exposure \( \eta = 1,000,000 \).
- Distress exposure \( \hat{Q}_f \) and pervasiveness \( \hat{b} \) vs. \( \hat{Q}_n \).

Lines: (P)ooled OTC; (S)eparated OTC; (C)CP

\( P – S \): Envelopes of distress exposure, pervasiveness

\(^7\)Price impact parameters are as in Almgren and Chriss (2001).
Large Bankruptcies: Example of Market Impact

- Suppose $\hat{Q}_n = 10,000,000$; GARCH variance decay of 0.9.
- For CCP market:
  - Expected market impact: $-30$.
  - Effective annual volatility goes from 30% to 38%.
- If pooled OTC buyers, sellers overtrade $1.75 \times$ at $t = 1$.
  - Annual volatility $\uparrow$ to 328% (instant.), 146% (effective).
- If OTC buyers and sellers separate, at $t = 1$:
  - Expected market impact: $-41$.
  - Annual volatility $\uparrow$ to 596% (instant.), 268% (effective).
Large Bankruptcies: Example of Real Effects

- Suppose $\hat{Q}_n = 10$ MM, market size of $40$ MM\(^8\).
- If 8% equity premium and mean risk aversion of $\hat{\lambda} = 3$:
  - Equilibrium allocation to risky asset: 29% (71% cash).
  - Post-crisis: 19% (CCP), 1.2% (OTC pool), 0.4% (OTC sep).
- Cost of distress externality:
  - $3.2$ MM (CCP), $123$ MM (OTC pool), $425$ MM (OTC sep).
  - Cost of OTC market distress is 3–11× market size.
- Given 2–3 bankruptcies; mean employees, compensation:
  - 260,000–400,000 unemployed; $33–$49 billion pay loss.
  - At 40% total taxes: revenue loss of $13–$20 billion.
- Also affects credit markets, overall macroeconomy.

\(^8\)Approximately $2(\hat{Q}_n + \hat{Q}_f)$. 
Complete networks admit two destabilizing events:
- Checkmate: weak counterparty may have no beneficial trade.
- Hunting: counterparties force others into bankruptcy.

Worse, hunting is a full equilibrium behavior.
- Market may be pushed far beyond one follow-on bankruptcy.

Are counterparties selfishly amoral/evil? Maybe not.
- Trade amount may pre-hedge expected follow-on bankruptcies.
- This reduces surprise need for trading in period 2.

CCP markets have fewer such destabilizing events.
- Suggests central clearing reduces OTC market volatility.
Difference from Allen and Gale (2000)

- Allen and Gale (2000): complete networks are more robust.
- I disagree: complete networks are more fragile.
- Allen and Gale approach: top-down.
  - Net exposure: $Q_i \sim N(0, (n-1)\eta^2)$
  - Contract exposure: $q_{ij} = Q_i/(n-1)$. (all same sign)
- My approach: bottom-up.
  - Contract exposure: $q_{i<j} \sim N(0, \eta^2); q_{ij} = -q_{ji}$
  - Net exposure: $Q_i = \sum_{j\neq i} q_{ij}; Q_i \sim N(0, (n-1)\eta^2)$.
- Same net exposures $Q_i$’s, different contract exposures $q_{ij}$’s.
- Strategic separation of buyers, sellers unlikely in A&G.
Conclusion

- Even small bankruptcies temporarily increase volatility.
- For a large bankruptcy in a bilateral OTC market:
  - Counterparties may be unable to save themselves (checkmate).
  - Counterparties may hunt their weakest peers for profit.
  - Volatility externality (and thus cost) higher than CCP market.
- Self-segregating buyers, sellers in OTC markets can be nasty:
  - Externality distress cost $\gg$ market size. (market failure?)
- Suggests benefits to centralized clearing in OTC markets\(^9\).
- Volatility externality cost $\Rightarrow$ when to move markets to CCP.
- May be able to measure when markets are more/less brittle.
  - $n$, $\eta$, $\bar{K}$ for part of market like complete network.

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\(^9\)Biais, Heider, Hoerova (2011) suggests CCP is capital efficient.