Valuing Financial Assets with Liquidity Discount: An Implication to Basel III

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Abstract

The unprecedented financial crisis in 2007 and 2008 and the largest bankruptcy in U.S. history prompted expedited regulation in the financial industry. A new Basel Accord has been proposed to further regulate the main risk that caused the crisis: liquidity risk. In a recent article, Chen [2012] presents a liquidity discount model in which financial securities can be evaluated with substantial discounts at the presence of a liquidity squeeze in the market place. In this article, we adopt this model to evaluate a selection of the 23 largest U.S. financial institutions (assets over $100 billion) to investigate the liquidity impact during the crisis period. We calibrate the model to market information such as market capitalization and volatility. We find that the model can provide significant predictive power of a bank’s liquidity health.
Valuing Financial Assets with Liquidity Discount:
An Implication to Basel III

The unprecedented financial crisis in 2007 and 2008 and the largest bankruptcy in U.S. history prompted quick passage of financial industry regulation. At the international level, a new Basel Accord has been proposed to further regulate the main risk that caused the crisis—liquidity risk. The consultative documents entitled “Strengthening the Resilience of the Banking Sector” and “International Framework for Liquidity Risk Measurement, Standards, and Monitoring” are a part of the Basel Committee’s ongoing work in response to the crisis. These two documents mention liquidity risk specifically as part of the Basel III regulation.

In looking at banks’ liquidity health, it is important to understand the definition of default because of the impact defaults can have on financial liquidity. Financial economists and accountants have different concerns regarding a firm’s default risk. Financial economists are concerned with no-arbitrage pricing and hence define a firm’s default by whether a firm has sufficient assets to pay for all of its debts. In other words, a firm must have a positive liquidation value to survive, or it must be in default. Accountants, on the other hand, are concerned only with whether a firm has enough liquid assets to pay for its short-term (within one year) liabilities. The former is “economic default,” and the latter is “liquidity default.”

By definition, the total value of all debts must exceed the value of the next-period debt. As a result, in usual situations (in which a firm can easily liquidate any portion of its assets), the firm should face an economic default before it faces a liquidity default. In other words, if a firm is under the threat of liquidity default, then it could have already defaulted under economic terms. In a “squeezed” environment (such as the recent financial crisis), however, assets cannot be liquidated easily and therefore suffer from huge liquidity discounts, which deviate substantially from the fundamental values of those assets. As a result, valuable assets are sold at large discounts because firms are forced to liquidate such assets quickly in order to pay for short-term liabilities, and the liquidity default may come before the economic default.  

1 Many of the assets recovered handsomely after the crisis. The Fed announced its large profits in its holding of mortgage-backed securities. The decision to purchase large volumes of assets through March 2010 came in two steps. In November 2008, the Federal Reserve announced purchases of housing agency

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In a recent article, Chen [2012] presents a liquidity discount model in which financial securities can be evaluated with substantial discounts at the presence of the liquidity squeeze in the marketplace. Combining the Geske [1977] and Chen [2012] models, this article provides empirical analysis of financial institutions’ asset values. By adopting both models and using market information to calibrate the model, this study can evaluate the liquidity impact on the banks during the crisis period. We find that the model can provide significant explanatory power of a bank’s liquidity health.

In this article, we use the Geske [1977] model to describe the capital structure of a firm. The economic and liquidity defaults defined earlier are formalized under the Geske model. We then combine the liquidity model by Chen [2012] with the Geske model to study a financial institution’s asset value. As a result, regulators can use the model developed in this article to monitor the liquidity condition for the entire banking industry once all the banks in the industry are included. Furthermore, each bank can also adopt this model to understand its own liquidity risk and adopt necessary steps to enhance its liquidity (in compliance with the Basel regulation) before it is too late.

In the next section, we describe the combined model used for banks’ assets. We then follow with empirical work that studies 23 U.S. banks.

A MODEL FOR BANKS’ ASSETS UNDER A LIQUIDITY SQUEEZE

This section presents a model to evaluate a bank’s assets under a liquidity squeeze. This model combines the liquidity discount model by Chen’s [2012] liquidity discount model with Geske’s [1977] capital structure model. To facilitate implementation of the combined model, this study adopts the assumptions made separately by the two models. This approach, however, does create minor consistency issues (to be addressed in more detail later), but the empirical results should be robust nonetheless.

Debt and agency mortgage-backed securities (MBS) of up to $600 billion. In March 2009, the Federal Open Market Committee (FOMC) decided to substantially expand its purchases of agency-related securities and to purchase longer-term Treasury securities as well. With total asset purchases of up to $1.75 trillion, total Federal Reserve assets doubled compared to total assets prior to 2008. The FOMC stated that the increased purchases of agency-related securities should “provide greater support to mortgage lending and housing markets” and that purchases of longer-term Treasury securities should “help improve conditions in private credit markets” [Federal Reserve Bank of New York, 2011]. The Treasury Department also established a program to purchase agency MBS beginning in September 2008. By the program’s termination at year-end 2009, it had purchased $220 billion of such securities. This program was much smaller than the Federal Reserve’s Large-Scale Asset Purchases (LSAPs), and no specific purchase amount targets were announced, so our analysis does not include this program.
The Model of Liquidity

This subsection provides a brief overview of the Chen [2012] model. To evaluate the discount (or premium) caused by illiquidity, one needs a convex (or concave) relationship between the asset value ($A$) of the firm and the fundamental economy (proxied by wealth, $W$). The liquid asset value is computed by the binomial model of Cox, Ross, and Rubinstein (CRR): At a given future date ($T$), the asset value is convex in wealth. In this article, we specify a call payoff to capture such convexity. That is, we specify that $A_T = \max\{W_T - K, 0\}$, where $K$, the strike price of the call, represents the convexity of the payoff. In Chen, the underlying wealth variable, $W_T$, is assumed to follow a lognormal distribution with mean $\mu_W$ and variance $\sigma^2_W$.

Chen argues that the larger the convexity, the larger the discount. As a result, the “liquidity health” of the assets ($A$) is determined by $K$. When $K = 0$, illiquidity has no impact on the asset value $A$. Later on, we calibrate this parameter to the firm’s spreads.

According to Chen, when no trading is permitted for the asset, the illiquid price is computed by the following equation:

\[
A^*_t = \frac{1}{R(t, T)} \left[ \mathbb{E}[A_T] - \beta^8 \{ \mathbb{E}[W_T] - R(t, T)W_t \} \right],
\]

where $R(t, T) = e^{(T-t)}$ is the risk-free money market account and $\beta^8 = \frac{\text{cov}[A_T, W_T]}{\text{var}[W_T]}$ is known as the dollar beta. This solution must be computed numerically. Exhibit 1 is generated using the following parameters:

<table>
<thead>
<tr>
<th>$K$</th>
<th>$r$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$T - t$</th>
<th>No. of Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>80, 100, 120</td>
<td>5%</td>
<td>0.6</td>
<td>0.8</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>

where the liquid price (plotted vertically) is computed using the CRR binomial model and the illiquid price is computed using Equation (1) (plotted horizontally).

[Exhibit 1 Here]

In Exhibit 1, the horizontal axis represents the illiquid value of the asset (symbolized by $A^*$), and the vertical represents the liquid value (symbolized by $A$). The 45-degree line represents perfect liquidity, in which there is no difference between the

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2 We also tried put payoff, and the result is similar. As Chen [2012] points out, any convex function is good enough to generate a liquidity discount.

3 The “$K$” parameter is set to 0.
liquid and the illiquid values. The solid curve, dotted curve, and the dash-dotted curve represent different magnitudes of the convexity parameter, \( K \). As Exhibit 1 illustrates, the greater the convexity, the greater the discount.

**Economic Default and Liquidity Default**

Let the asset (only one class to begin with) of a financial company follow the Black–Scholes model:

\[
\frac{dA}{A} = \mu_A dt + \sigma_A dz,
\]

where \( z \) is Brownian motion and \( \mu_A \) and \( \sigma_A \) are drift and diffusion, respectively. For simplicity, we assume that the firm has two debts (to be extended to multiple-coupon debts)\(^4\), both zero coupon, with face values \( K_1 \) and \( K_2 \) and maturities \( T_1 \) and \( T_2 \), respectively.

When the economy is “normal” and the market is perfectly liquid,\(^5\) it must be the case that defaults can occur only as the result of economic reasons. In the Merton [1974] and Geske [1977] models, “economic default” is defined as \( A_{T_1} < \bar{A}_T \), where \( \bar{A}_T \) represents the total value of debts (see Geske for details). The equity value under this case, presented by Geske, is given by

\[
E_i = A_i N_2 \left( d_1^+, d_2^+; \sqrt{1 - r^2}, -e^{-r(T_1-t)}K_1N_1(d_1^-) - e^{-r(T_2-t)}K_2N_2(d_1^-), d_2^-; \sqrt{1 - r^2} \right),
\]

where \( N_2(x_1, x_2; \rho) \) is the bivariate normal probability function with limits \( x_1 \) and \( x_2 \), correlation \( \rho \), and

\[
d_i^\pm = \frac{\ln A_t - \ln X_i + (r \pm \sigma_i^2)(T_i - t)}{\sigma_i \sqrt{T_i - t}}
\]

\[
X_1 = \bar{A}_{T_1}
\]

\[
X_2 = K_2
\]

When the economy is under liquidity stress, the asset value is compressed. For this scenario, we represent the liquidity-compressed price as \( a \times A = A^* \), where \( 0 < a < 1 \).

As a result, the liquidity default is defined (going concern) as \( A_{T_1}^* < K_1 \). The equity value then is given by

\[
E_i^* = e^{-r(T_1-t)E_{T_1} \left[ E_{T_2} \left( \max\{A_{T_1}^* - K_1, 0\}, K_2 \right) \right]}
\]

\(^4\) But under a specific seniority order, as Geske and Johnson [1984] assume.

\(^5\) The day-to-day usual and minor liquidity discounts are assumed away here.
where $E_{T_1}(\max\{A_{T_1}^* - K_1, 0\}, K_2)$—that is, the equity value at time $T_1$—is a call valuation with $\max\{A_{T_1}^* - K_1, 0\}$ as the underlying asset value and $K_2$ as the strike price.

Equation (3') indicates that in a state where the firm survives, it must be that the firm has enough assets $A_{T_1}^*$ to pay for its current debt $K_1$. If so, the debt is paid for by the assets, and the firm’s assets reduce to $A_{T_1}^* - K_1$. As a result, the Geske model will price the equity using $A_{T_1}^* - K_1$.\(^6\)

Equation (3') can be implemented only numerically. In the empirical work, $A_{T_1}^*$ is approximated as follows:

$$ A_{T_1}^* = A_t^* \exp \left( -\frac{1}{2} \sigma_A^2 + \sigma_A \int_T^{T_1} dz \right) $$

which is equivalent to the lognormal process without drift. Note that $A_t^*$ is computed using Equation (1). The equity value at time $T_1$, $E_{T_1}$, is carried out using the Black–Scholes model by substituting $\max\{A_{T_1}^* - K_1, 0\}$ for the underlying asset value. To carry the call values at $T_1$ back to $t$ to arrive at $E_t^*$, the standard binomial model with 100 steps is used.\(^7\)

**Modeling Assets**

With the liquidity discount model and the capital structure model established, it is possible to now value a bank’s assets. We continue to use the parameters with a choice of $K = 80$ in the previous subsection for the Chen model to compute the illiquid asset value $A_t^*$. For the capital structure model, we assume the following:

$\begin{align*}
K_1 & \quad K_2 & \quad \sigma_A & \quad r & \quad T_1 & \quad T_2 \\
50 & \quad 50 & \quad 0.3 & \quad 0.05 & \quad 1 & \quad 2
\end{align*}$

Note that for every given $A_t$, we use the economic default model—Equation (3)—to compute the liquid equity value $E_t$. At the same time, we use the liquidity discount model (demonstrated in Exhibit 1) to compute the illiquid asset value $A_t^*$.\(^8\) Then Equation (3') is used to compute the illiquid equity value $E_t^*$.

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\(^6\) Given that this is only a one-period calculation, the Geske model is identical to the Merton model.

\(^7\) Note that in the empirical work, the illiquid asset value is assumed to follow the same lognormal distribution as the liquid asset value.

\(^8\) One needs to infer the wealth value $W$ from the CRR binomial model and then compute the illiquid asset value using Equation (1).
Exhibit 2 plots $E_t^r$ and $E_t$ against the liquid asset value, $A_t$. In Exhibit 2, for each liquid asset value, we use Exhibit 1 to map out the illiquid asset value. Then we use the binomial model to compute the liquid equity value (solid line) and illiquid equity value (dotted line). In the gray area, the market is liquid and the “economic equity value” is lower (as argued before, $E_t^r > E_t^r$). In the yellow area, the market is under a liquidity squeeze, and hence the “liquidity equity value” is lower. Using the liquidity discount model in Chen, we find that the two curves cross, as shown in the figure.

[Exhibit 2 Here]

We argue that equity investors will price the equity using the economic default model when there is no liquidity concern and using the liquidity default model whenever there is liquidity concern. As a result, the equity value is the smaller of the two economic/liquidity values. The crossover point in Exhibit 2 separates economic default from liquidity default. The left side of the crossover point represents situations in which liquidity defaults dominate, and the right side of the crossover point represents situations in which economic defaults dominate.

Typically, it is unknown whether an equity value observed in the marketplace reflects the liquidity value or the economic value of the assets. The model portrayed in Exhibit 2 allows for this distinction. If a firm is dominated by the risk of a liquidity default, then the model will suggest a higher asset value when using Equation (3) than when using Equation (3*). Similarly, if a firm suffers no liquidity problems (i.e., dominated by the risk of an economic default), the model will suggest a higher asset value when using Equation (3*) than when using Equation (3). This distinction facilitates the empirical study of banks’ liquidity health.

In the next subsection, we use the market equity value (market capitalization) to infer the asset value assuming perfect liquidity (i.e., using Equation (3)). In other words, we assume the market equity value as the economic value (red line in Exhibit 2) and compute the liquid asset value $A_t$ (horizontal axis in Exhibit 2). Then we use the Chen model to compute the illiquid asset value, $A_t^r$, and the illiquid equity value, $E_t^r$. The convexity parameter in the Chen model, $K$, is solved by calibrating the model to the

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9 A similar graph can be plotted against the illiquid asset value, $A_t^r$.
10 To do this, note that each liquid price of the asset is mapped to a wealth level, and then Equation (1) is used to calculate the illiquid asset value.
firm’s credit spreads. The results, discussed in the next subsection, show that during a crisis period, the illiquid values deviate substantially.

**Discussions**

To take advantage of the closed-form solution of the Geske model for the capital structure of a firm, the firm’s asset value must follow a lognormal distribution. Chen’s liquidity discount model, however, will not necessarily generate a lognormal distribution for the asset value. In fact, the functional form of the Chen model used in this article is the same as the one used in the original study, where the underlying state variable (wealth) follows a lognormal distribution and hence the resulting asset value is not a lognormal distribution.

Consequently, the empirical results we obtain in this article are only approximations. We argue that these approximations do not change our conclusion qualitatively, however, in that 1) calibration cancels many of the approximation errors and 2) we measure the liquidity impact only in a relative manner, and hence absolute magnitudes are not used.

We note that both distributions for the asset value (lognormal in the Geske model and a resulting non-lognormal in the Chen model) are right skewed, and discrepancies are less when the skewness level is lower.

**EMPIRICAL WORK**

We apply the model to examine the 23 largest banks in the United States to investigate how their assets are affected if the market faces a liquidity squeeze. We use market information—that is, market capitalization and its volatility—to infer the implied liquid and illiquid asset values. These values differ from the book value of assets in that they reflect the evaluations of equity investors. In other words, we assume that equity investors correctly evaluate the firm’s assets and credit risk (via its capital structure) and assign a value to the equity. When the market is free from a liquidity squeeze, then the equity value should reflect the perfectly liquid asset value. Similarly, when the market is under a liquidity squeeze, the asset value is compressed and the equity value is also lowered to reflect the liquidity-discounted asset value.

As a result, we adopt the following steps to calibrate the model to the market information:
We compute, monthly, $K_1$ and $K_2$ values (the data contain monthly Lehman cash flows, but we aggregate the cash follows to be annual) paid at $T_1$ and $T_2$ respectively;\(^\text{11}\)
- We compute the bank’s market capitalization and its volatility monthly;
- We solve for $A_t$ by substituting the market capitalization for $E_t$ using Equation (3);\(^\text{12}\)
- We use the Chen model (i.e., Exhibit 1) to compute $A_t^*$ by calibrating $K$ to the credit spreads of Lehman; and
- We compute $E_t^*$ using $A_t^*$.

Lehman Brothers Inc. serves as an example (case study) to explain in detail how the calibration of the model works. We then study the remaining 23 banks for their liquidity-discounted asset values.

**Data**

The data we use in this study contain Lehman Brothers and a set of the 23 largest banks in the United States during the period from January 2004 to December 2009. This period covers the peak of the real estate bubble and the financial crisis trigged by the Lehman default. The debt data, obtained from FactSet, include all the liabilities issued by the banks. Using the coupon, maturity, and other specific information (e.g., floating/fixed, call/put provision, amortization, and other miscellaneous items), we estimate the cash flows and bucket them into monthly amounts (month-end). In the empirical work, however, we further group these numerous cash flows into two cash flows $K_1$ and $K_2$. In the empirical work, we sum up all the first-year cash flows as $K_1$. To calculate $K_2$, we sum up all the second-year cash flows and then add half of all the remaining cash flows.

The equity data are obtained monthly (month-end) from Yahoo.com and used to compute the equity volatility and market capitalization. The outstanding shares are obtained from the annual reports (to compute market capitalization). The risk-free rate used in the corporate finance model (Equations (3) and (3*)) is the three-month Treasury rate taken from Bloomberg.

\(^\text{11}\) We follow the KMV method, in which all cash flows after $T_2$ are aggregated and halved and then added to the $T_2$ cash-flow.

\(^\text{12}\) The asset values computed with this method are the liquidity values.
Lehman Case Study

We use Lehman Brothers Inc. as an example to describe in detail how we calibrate the model to estimate the liquidity discount of Lehman’s asset value. We report the liquidity-discounted asset values during the sample period and demonstrate the inevitability of Lehman’s default.

On September 15, 2008, Lehman Brothers Holdings Inc. (Lehman hereafter) led the largest bankruptcy in the U.S. history at a total of $138 billion. Prior to its bankruptcy, Lehman was the fourth-largest investment bank in the United States behind Goldman Sachs, Morgan Stanley, and Merrill Lynch.

At the time of its failure, Lehman was highly leveraged and used a large amount of short-term repurchase transactions (also called repos). The high leverage and reliance on short-term financing was rumored to have led to difficulties in Lehman being able to renew the contracts, and banks refused to lend to Lehman. Lehman's fall marks the beginning of the credit crisis and the worst economic recession since World War II.

Whether Lehman's default, and indeed the entire crisis, was a liquidity crisis or a credit crisis is an ongoing debate. Lehman reported earnings of $489 million for the first quarter of 2008 and was able to raise $4 billion of equity capital in April. But this turned out to be too little and too late. Our results indicate that Lehman’s liquidity started to deteriorate in mid 2007 as Bear Stearns revealed its troubles in the two hedge funds and has not recovered since. Exhibit 3 shows a time line of events at Lehman.

Exhibit 3 displays the debt maturity structure from December 2007 to September 2008. The graph shows the notional debt value maturing in each year. As the exhibit illustrates, short-term debt maturing in one to three years dominated in Lehman's liability structure. Such a liability structure is typical for financial institutions that finance their operations using liquid, short-term debt. Exhibit 4 also shows a spike for debt maturing after 30 years, which includes perpetual debt, preferred securities, and 30- to 40-year mortgage-backed securities. It is very important to note that that Lehman’s short-term debt increased dramatically after March 2008. This increase reflects the constraints imposed on Lehman after the fall of Bear Stearns. We are able to show, however, that these short-term debts put even more pressure on Lehman as the financial crisis worsened.
To estimate Lehman’s asset values, we first simplify the debt structure to have only two annual payments to fit to Equation (3). The first payment, $K_1$, equals the first cash flow due in one year (seen in Exhibit 4). The second payment, $K_2$, equals the second cash flow plus half of all the remaining cash flows.\footnote{This method is proposed by KMV.}

The market capitalization is used as the equity value in the Geske model—Equation (3). The volatility, $\sigma_A$, is estimated using daily one-year historical continuously compounded stock returns. The volatility of stock returns is an equity volatility, $\sigma_E$, and needs to be translated to the asset volatility, $\sigma_A$, with the transformation formula $\sigma_A = \sigma_E (E/A) \Delta$, where $\Delta = N_2\left(d_1^+, d_2^+; \sqrt{4 - 1}\right)$, defined in Equation (3). Now we can proceed to estimate the asset value (liquid) of Lehman using the market capitalization value as the equity value. This approach (of using market cap and the volatility of market cap) to solve for the asset value and asset volatility is adopted widely in industry and academic research.

After solving for the asset value and asset volatility, we compute the liquidity-constraint asset value using the Chen model. Several parameters in this model are preset: the frequency of rebalancing (symbolized by $k$ in the original article) is set to 0; the mean and standard deviation of the underlying state variable $\mu_w$ and $\sigma_w$ are set to 0.6 and 0.3, respectively; the risk-free rate $r$ is set to 0; and finally, the number of steps for the binomial model is set to four, to conserve time.\footnote{None of these parameter values has any material impact on the final result as we calibrate the model to the market information. Currently we use only information from equities. Should more information be available for calibration, many of these parameters can be estimated more meaningfully.} The number of steps used in implementing Equation (3*) is 100.

Now we are left with only one parameter, $K$, which represents the convexity of the liquidity discount function. To minimize the calibration, we use the “implied credit spreads” from the Geske model (see the Appendix for the spread calculation). Because liquidity and credit risks are highly correlated (see, e.g., a recent study by Imbierowicz and Rauch [2012]), this calibration is reasonable.\footnote{We could calibrate the parameter to the CDS spreads and achieve similar results because of the extremely high correlation between the implied credit spreads and the CDS spreads. The result is available on request.} Because liquidity worsens when the spread widens, we set the parameter to $K = W \times (1 - 4(10\% - s))$, where $W$ is wealth and $s$ is the implied spread. The two scalars, 4 and 10\%, are designed to bring the level of the
spread in line with the level of the convexity parameter. These two scalars only parallel-shift the illiquid values from the liquid values and do not change the relative relationship between them. One alternative method to estimate these two scalars is to use cross-sectional data, which is beyond the scope of this article.

Note that $A' \leq A$ by construction, as demonstrated clearly in Exhibit 1. As a result, during a “normal” time, the equality holds, and during an “illiquid” time, the inequality holds. Exhibit 5 plots the liquid asset value ($A$) and the illiquid asset value ($A'$) of Lehman from January 2004 until its bankruptcy in August 2008. In Exhibit 5, the solid line represents the liquid asset value and the dotted line represents the liquidity-constrained asset value.

[Exhibit 5 Here]

In the case of Lehman, during the 2005–2006 period, there was no liquidity squeeze and hence the two lines were together. Before 2005, there was slight liquidity squeeze, and starting in 2008 the liquidity issue became quite severe and eventually led to bankruptcy. The model is able to predict Lehman’s default six months in advance.

**Other Examples**

Exhibit 6 shows the sample period January 2004 to December 2009 in months, with January 2004 as month 1 and a total of 72 months. Month 49 is January 2008, which is right before the Bear Stearns default; months 13 to 36 represent 2005 and 2006, when the real estate bubble peaked; and month 57 is September 2008, when Lehman defaulted.

[Exhibit 6 Here]

The findings can be broken into three different groups. The first group consists of liquidity-healthy banks: BBT, STT, BK, TRV, BRK.A, and PNC. The second group consists of banks that were healthy until the crisis occurred (month 58): PRU, USB, STI, COF, FITB, ALL, PFG, and AIG. This group of banks was deeply affected (spillover) by the Lehman bankruptcy. The third group of banks demonstrated early signs of liquidity weakness: WAMUQ, C, BAC (not so early), SLM, GS, FRE, FNM, GNW, and AXP. These banks were similar to Lehman in their liquidity vulnerability.

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16 The appendix lists the banks’ full names.
Out of the 23 banks in the sample, FNM (Fannie Mae) and FRE (Freddie Mac) are the two poorest performers. Throughout the sample period, they never demonstrated enough liquidity, even during the peak of the bubble. This result is striking in the sense that they were more highly rated by rating agencies than all the other banks in the sample, likely because of implicit guarantees by the government.

Another point deserving special mention is that GS (Goldman Sachs) was the only bank (except for FNM and FRE) that demonstrated liquidity weakness at the time (month 42, or July, 2007) when Bear Stearns’ two troubled hedge funds unfolded. Because GS is the largest investment bank, its liquidity weakness at the beginning of the crisis signals that the whole investment banking industry may be vulnerable to liquidity risk.

**CONCLUSION AND EXTENSION**

In this article, we combine Chen’s [2012] liquidity discount model and Geske’s [1977] corporate finance model to estimate the asset value of the 23 largest U.S. banks. We discover that in almost no circumstance between 2004 and 2009 were these banks dominated by economic default. Rather, they were affected by liquidity default.

Furthermore, in hindsight (in-sample test), our model is quite predictive in the liquidity discounted value of the assets. As a result, the model we propose in this article is a reasonable tool for banks and regulators to use in monitoring the liquidity health of either individual financial institutions or the entire economy.

A natural extension of our model can be used to study liquidity discounted value for various asset classes. In this article, we calculate only one asset value—the aggregated value of all of a firm’s assets. To provide better risk management, it is conceivable to provide valuation for each of the distinctive asset classes (e.g., Treasuries versus mortgage-backed securities).
REFERENCES


Exhibit 1. Relationship between Liquid Price (A) and Illiquid Price (A*) under Various Convexity Levels

Note: The plot depicts the relationship between liquid asset value (A) and illiquid asset value (A*). The state variable is assumed to be lognormally distributed with $\mu_w = 0.6$, $\sigma_w = 0.3$. The liquid price is computed by the CRR binomial model with $r = 5\%$, $T - 1 = 1$, and $N = 100$. The illiquid price is computed by Equation (1).
Exhibit 2. Equity Values under Liquidity and Economic Defaults

Note: The plot depicts equity values under economic default, Equation (3), and liquidity default, Equation (3*). In addition to the parameters used in Exhibit 1 for the liquidity discount model, the capital structure parameters are $K_1 = 50$, $K_2 = 50$, $\sigma_A = 0.3$, $r = 5\%$, $T_1 = 1$, and $T_2 = 2$. 
Exhibit 3. Lehman Timeline

2007 Q4: Lehman shows $886 million in quarterly earnings

3/14: Lehman obtains a $2 billion, three-year credit line from a consortium of 40 banks

3/16: JPMorgan announces a deal to purchase Bear Stearns for $2 per share

4/1: Lehman looks to raise $4 billion in new capital via an offering of perpetual convertible preferred stock.

6/9: Lehman announces plan to raise an additional $6 billion

6/9: Lehman shows a $2.8 billion loss, the first loss in its history as a public firm.

9/9: Markets punish Lehman for not raising capital more aggressively; Lehman’s share price falls 45%

9/15: Lehman officially files bankruptcy

2007 to January 2008: Lehman scales back its mortgage business, cutting thousands of mortgage-related jobs and closing mortgage origination units

9/22/2008
Exhibit 4. Lehman Debt Structure

Exhibit 5. Liquid and Illiquid Asset Values of Lehman
Exhibit 6. Liquid and Illiquid Asset Values of the 23 Largest U.S. Banks
Panel A. Safe Banks
Panel B. Lehman Spillover Banks
Panel C. Early Warnings

Graphs showing financial performance of various companies from Jan-04 to Sep-09.
APPENDIX

List of 23 Banks’ Full Names

AIG  American International Group, Inc.
ALL  The Allstate Corporation
AXP  American Express Company
BAC  Bank of America Corporation
BBT  BB&T Corporation
BK   The Bank of New York Mellon Corporation
BRK.A Berkshire Hathaway Inc.
C    Citigroup, Inc.
COF  Capital One Financial Corp.
FITB Fifth Third Bancorp
FNM  Federal National Mortgage Association
FRE  Freddie Mac
GNW  Genworth Financial Inc.
GS   The Goldman Sachs Group, Inc.
PFG  Principal Financial Group Inc.
PNC  PNC Financial Services Group Inc.
PRU  Prudential Financial, Inc.
SLM  SLM Corporation
STI  SunTrust Banks, Inc.
STT  State Street Corporation
TRV  The Travelers Companies, Inc
USB  U.S. Bancorp
WAMUQ Washington Mutual Inc.

Geske’s Implied Spread Calculation

Note that in addition to the equity value, we can also derive pricing formulas for all the bonds (in this case, $K_1$ and $K_2$) of a bank as follows:

\[
D_{t,T_1} = e^{-r(T_1-t)}K_1 N_1(d_1^-) + A_t[1 - N_1(d_1^+)]
\]

\[
D_{t,T_2} = e^{-r(T_1-t)}K_2 N_2(d_1^-, d_2^-; \rho) + A_t[N_1(d_1^+) - N_2(d_1^+, d_2^+; \rho)]
\]

As a result, credit spreads can be computed as $s_{t,T_1} = r(T_1 - t) - \ln[1/D_{t,T_1}]$, where $r$ is the risk-free rate.
**Lehman Timeline**

2007 to January 2008: Lehman scales back its mortgage business, cutting thousands of mortgage-related jobs and closing mortgage origination units.

2007 Q4: Lehman shows $886 million in quarterly earnings (at compared to third quarter) and reported earnings of $4.192 billion for fiscal year 2007 (a 5% increase from the previous fiscal year).

January 29, 2008: Lehman announces an increase in dividends and plans to repurchase up to 100 million shares of common stock.

2008 Q1: Lehman increases holding of Alt-A mortgages despite the prevailing troubles in the real estate market.

March 14, 2008: Lehman obtains a $2 billion, three-year credit line from a consortium of 40 banks, including JPMorgan Chase and Citigroup. On the same day, the Federal Reserve and JPMorgan Chase begin to put together a deal to bail out Bear Stearns.

March 16, 2008: JP Morgan announces a deal to purchase Bear Stearns for $2 per share.

March 18, 2008: Lehman shares surged up almost 50% after the Federal Reserve gives investment banks access to the discount window.

April 1, 2008: Lehman looks to raise $4 billion in new capital via an offering of perpetual convertible preferred stock.

2008 Q2: Lehman shows a $2.8 billion loss, the first loss in its history as a public firm. It admits the losses came not only from mortgage-related positions but also from hedges against those positions.

June 9, 2008: Lehman announces plan to raise an additional $6 billion in new capital ($4 billion in common stock, $2 billion in mandatory convertible preferred stock).

July 7 to July 11, 2008: Lehman shares plunge more than 30% for the week amid rumors that the firm’s assets have not been priced appropriately to reflect the true value.

September 9, 2008: Markets punish Lehman for not raising capital more aggressively; Lehman’s share price falls 45% to $7.79 on fears that the firm’s capital levels are insufficient to support exposure to deteriorating real estate investments.

September 10, 2008: Lehman CEO Dick Fuld reveals plans to spin off real estate assets and sell a portion of the asset management division, insisting that the firm is solvent enough to survive.

September 11, 2008: Talks of a Lehman takeover permeate the markets as Lehman shares fall further, closing at $4.22.

September 12, 2008: Lehman approaches several potential buyers, including Bank of America and Barclays.

September 15, 2008: Lehman officially files bankruptcy after Treasury Secretary Paulson refuses to back any takeover; Shares close at $0.21.

September 16, 2008: Lehman is dropped from the S&P 500 Index.

September 18, 2008: Lehman shares close at $0.052 in over-the-counter trading as effects of the biggest bankruptcy in history ripple through the financial markets.

September 22, 2008: Lehman’s U.S. operations reopen for business under Barclays Capital after approval for the acquisition was granted by the federal bankruptcy court presiding over the liquidation.